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Letter to the Editors

Comment on the paper by Qi-Hong Deng, Guang-Fa Tang, "Numerical visualization of mass and heat transport for conjugate natural convection/heat conduction by streamline and heatline" IJHMT 45 (11) (2002) 2373–2385

In a recent paper [1] the use of streamlines and heatlines for visualization purposes is considered, and specially their use in conjugated conduction/convection heat transfer problems. Special emphasis is devoted to the treatment of the diffusion coefficient for the heatfunction, using a so-called 'consistent formulation'. The main difference on this work [1] relative to a previous work on the subject [2] relies on the treatment of such diffusion coefficients. It is claimed that the diffusion coefficient for the heatfunction must be invariant over the solid and fluid portions of the domain, taking the value of unity over all domain, contrarily to what is explained in [2]. The present comment is centered on the definitions of the diffusion coefficients for the heatfunction (and also for the streamfunction the massfunction, as the main goal is a completely unified treatment), where it is shown that such diffusion coefficients are not constant over all the domain, contrarily to what is claimed in [1] and in complete accordance with [2].

Starting from the steady general conservation equation for ϕ written as

$$\frac{\partial}{\partial x}\left[\rho u(\phi-\phi_0)-\Gamma_\phi\frac{\partial\phi}{\partial x}\right]+\frac{\partial}{\partial y}\left[\rho v(\phi-\phi_0)-\Gamma_\phi\frac{\partial\phi}{\partial y}\right]=0 \tag{1}$$

function $\Phi(x,y)$ is defined through its first-order derivatives

$$\begin{aligned}\frac{\partial\Phi}{\partial y}&\equiv\rho u(\phi-\phi_0)-\Gamma_\phi\frac{\partial\phi}{\partial x}\\ -\frac{\partial\Phi}{\partial x}&\equiv\rho v(\phi-\phi_0)-\Gamma_\phi\frac{\partial\phi}{\partial y}\end{aligned} \tag{2}$$

Assuming that ϕ is a continuous function to its second-order derivatives, equality of the second-order cross derivatives, $\partial^2\phi/\partial x\partial y=\partial^2\phi/\partial y\partial x$ leads to the differential equation for Φ,

$$\begin{aligned}0=\frac{\partial}{\partial x}\left(\frac{1}{\Gamma_\phi}\frac{\partial\Phi}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{1}{\Gamma_\phi}\frac{\partial\Phi}{\partial y}\right)+\Bigg\{\frac{\partial}{\partial x}\left[\frac{\rho v}{\Gamma_\phi}(\phi-\phi_0)\right]\\ -\frac{\partial}{\partial y}\left[\frac{\rho u}{\Gamma_\phi}(\phi-\phi_0)\right]\Bigg\}\end{aligned} \tag{3}$$

where it is evident that the *natural* diffusion coefficient for Φ is $\Gamma_\Phi=1/\Gamma_\phi$. It should be noted that Φ is treated as a conserved variable, its solution being obtained following the same (conservative) procedures as for ϕ [1,2]. It is tempting to delete Γ_ϕ from Eq. (3), as it is made in [1]. However, this is not correct for a multi-component domain, with different diffusion coefficients Γ_ϕ, similarly to what happens when analyzing pure heat conduction in contiguous media of different thermal conductivities.

Due to their physical nature, ϕ and Φ must be continuous at the solid–fluid interface. From Fig. 1, at each point of the interface s it is

$$\phi_1=\phi_2 \qquad \Phi_1=\Phi_2 \tag{4}$$

It should be stressed that $\Phi_1=\Phi_2$ guarantees the conservation of ϕ through the interface.

At the solid–fluid interface s of Fig. 1, where only diffusive transfer is present

$$\mathbf{J}_\phi\cdot\mathbf{n}=-\Gamma_\phi\left(\sin\theta\frac{\partial\phi}{\partial x}-\cos\theta\frac{\partial\phi}{\partial y}\right)=-\Gamma_\phi\frac{\partial\phi}{\partial n} \tag{5}$$

$$\mathbf{J}_\phi\cdot\mathbf{s}=-\Gamma_\phi\left(\cos\theta\frac{\partial\phi}{\partial x}+\sin\theta\frac{\partial\phi}{\partial y}\right)=-\Gamma_\phi\frac{\partial\phi}{\partial s} \tag{6}$$

The conservation principle of ϕ implies that, at the interface,

$$-\Gamma_{\phi,1}\left(\frac{\partial\phi}{\partial n}\right)_1=-\Gamma_{\phi,2}\left(\frac{\partial\phi}{\partial n}\right)_2 \tag{7}$$

where is evident that $(\partial\phi/\partial n)_1\neq(\partial\phi/\partial n)_2$ if $\Gamma_{\phi,1}\neq\Gamma_{\phi,2}$.

A function with unequal first-order side derivatives at a point has no first-order derivative at such point, as well as it has no higher order derivatives. Thus, Eq. (3) can be used through each portion of the domain with its own diffusion coefficient, $\Gamma_\Phi=1/\Gamma_\phi$, and the different portions of the domain must be linked through a careful treatment of the interfaces. Eq. (1) can be obtained identically by equating the second-order cross derivatives of Φ, $\partial^2\Phi/\partial x\partial y=\partial^2\Phi/\partial y\partial x$. This is possible only if Φ is a continuous function to its second-order derivatives, and by the same reasons as explained above for ϕ, special care is needed when dealing with points located at the solid–fluid interface.

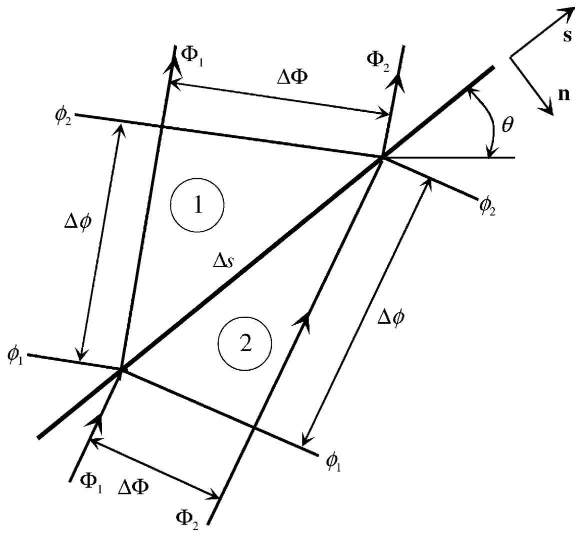

Fig. 1. Lines of constant ϕ and lines of constant Φ, normal to each other at any point, near the interface s between media 1 and 2 of different diffusion coefficients.

The remaining question is to analyze the consistency of the diffusion coefficient for Φ, $\Gamma_\Phi = 1/\Gamma_\phi$, when it is used together with the flux boundary conditions for ϕ and for Φ at the interface. The counterpart of Eq. (7) for Φ at the interface, taken as a conserved variable, is

$$-\Gamma_{\Phi,1}\left(\frac{\partial\Phi}{\partial n}\right)_1 = -\Gamma_{\Phi,2}\left(\frac{\partial\Phi}{\partial n}\right)_2 \qquad (8)$$

Similarly to Eqs. (5) and (6) for ϕ, it can be obtained for Φ that

$$\begin{aligned}\frac{\partial\Phi}{\partial n} &= \sin\theta\frac{\partial\Phi}{\partial x} - \cos\theta\frac{\partial\Phi}{\partial y}\\ \frac{\partial\Phi}{\partial s} &= \cos\theta\frac{\partial\Phi}{\partial x} + \sin\theta\frac{\partial\Phi}{\partial y}\end{aligned} \qquad (9)$$

Substituting $\partial\Phi/\partial x$ and $\partial\Phi/\partial y$ as given by Eq. (2) on the right sides of Eq. (9), it results the most general spatial form of Eq. (2) for the sole diffusive situation

$$-\frac{\partial\Phi}{\partial n} = -\Gamma_\phi\frac{\partial\phi}{\partial s} \qquad \frac{\partial\Phi}{\partial s} = -\Gamma_\phi\frac{\partial\phi}{\partial n} \qquad (10)$$

From Fig. 1 it can be observed that $(\Delta\Phi/\Delta s)_1 = (\Delta\Phi/\Delta s)_2$ and that $(\Delta\phi/\Delta s)_1 = (\Delta\phi/\Delta s)_2$. In the limit situation, when $\Delta s \to 0$, it results then

$$\left(\frac{\partial\Phi}{\partial s}\right)_1 = \left(\frac{\partial\Phi}{\partial s}\right)_2 \qquad \left(\frac{\partial\phi}{\partial s}\right)_1 = \left(\frac{\partial\phi}{\partial s}\right)_2 \qquad (11)$$

The first result of Eq. (11) (using the second result of Eq. (10)) was previously set by Eq. (7), invoking the conservation principle of ϕ at the interface.

The second result of Eq. (11) (now using the first result of Eq. (10)), states that

$$-\frac{1}{\Gamma_{\phi,1}}\left(\frac{\partial\Phi}{\partial n}\right)_1 = -\frac{1}{\Gamma_{\phi,2}}\left(\frac{\partial\Phi}{\partial n}\right)_2 \qquad (12)$$

Comparing this result with Eq. (8) it is evident that, in fact

$$\Gamma_\Phi = 1/\Gamma_\phi \qquad (13)$$

This confirms the diffusion coefficient $\Gamma_\Phi = 1/\Gamma_\phi$ as proposed and used in [2], contrarily to what was proposed and used in [1], $\Gamma_\Phi = 1$ over all domain. This is of crucial importance when dealing with different media with different (or even very different) diffusion coefficients. It is the unique approach that is consistent with the treatment of both the interior domain and the flux boundary conditions for ϕ and for Φ at the interface. When the main goal is a complete unified treatment of the streamlines, heatlines and masslines, $\Gamma_\Phi = 1/\Gamma_\phi$ proves also to be the unique, correct and effective way to do so [2].

References

[1] Q.-H. Deng, G.-F. Tang, Numerical visualization of mass and heat transport for conjugate natural convection/heat conduction by streamline and heatline, Int. J. Heat Mass Transfer 45 (11) (2002) 2373–2385.
[2] V.A.F. Costa, Unification of the streamline, heatline and massline methods for the visualization of two-dimensional transport phenomena, Int. J. Heat Mass Transfer 42 (1) (1999) 27–33.

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